



A Comparison of the Impact of some Basic Factors Influencing the Temperature of Saturated Sandy and Loamy soils

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Abstract

A comparison was made between the impacts that the internal heat and solar radiation have on temperature of saturated sandy & loamy soils. The problem was formulated using Boussinesq's approximation under some necessary conditions at which the heat transfer took place. The dimensional linear second order partial differential energy equation was first abridged to non-dimensional structure by using various regular dimensionless parametric quantities, and then to ordinary differential equation by perturbation technique. The emerged material parameters that comprise the Primordial heat and the solar radiation factor among related others were inspected on temperature of the two soil samples; sand & loam, saturated at the equivalent level of humidity. The results which were graphed showed that both factors enlarged the temperature of the two soils at different rates and levels.

Keywords: *Boussinesq's approximation, primordial heat, loamy soil, sandy soil, solar radiation.*



1.0 Introduction

In the research world, heat transfer problem have become renown because of its great importance and vast application. This cut across various field in applied sciences and engineering among others. Thus, Amrollah (2014) did a survey on thermal properties of Clay-Loam soil. His research was done in a laboratory using a single probe method in determining the thermal properties of this soil. He discovered that as the dampness of the soil is increasing, so are thermal properties increases. The same thing happens when the bulk density of the sand increases; the thermal properties are also shooting up. However, the change that occurred as a result of increasing bulk density is more than that of the moisture content.

Tong *et al* (2016) did an empirical study and developed a model that could be used to estimate thermal conductivity of soil from porosity and water content of a soil. The new model developed was said to be simpler than many existing models in this area and can also be incorporated into bigger models. Holmes *et al* (2008) also estimated the profile of soil temperature observed from one single depth. Their approach can be applicable in satellite-based energy balance.

Some others may include Oladunjoye & Sanuade (2012) who examined the specific heat of some soils alongside their thermal diffusivity at a Power Plant in Olorunsogo, Southwest Nigeria, Mahmoodi and Kianmehr (2008) that compared and determined thermal conductivity varieties

of pomegranate in Iran, Edem *et al* (2012) studied the effect that animal droppings have on the thermal properties on porous system that is dispersed, among others.

In view of these literatures and host of others, this study focused of the comparison of the impact of some basic factors influencing temperature of saturated sandy & loamy soils at the same wet basis using time-dependent thermal conductivity.

2. Mathematical Formulation

This study involves heat transfer with 2 dimensions. It was unsteady with Dirichlet boundary condition imposed. Taken to be infinite along horizontal axis (y'), and z' vertical axis which goes inside the soil, the flow area turn out to be functions of z' & t' only (Mohammed, 2013). Soil is considered to be an optically thin environment and solar radiation comes directly towards it in a direction of gravity. Following the propositions, leading equations under typical Boussinesq's approximation developed into:

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

Energy equation

$$\frac{\partial T'}{\partial t'} + w' \frac{\partial T'}{\partial z'} = \frac{1}{\rho C_p} \left[k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z} \right] + \frac{Q_0 (T' - T_\infty)}{\rho C_p} \quad (2)$$

Subject to:

$$T' = T'_w \text{ at } z' = 0 \quad (3)$$



$$T' \rightarrow T'_\infty \text{ as } z' \rightarrow \infty \quad (4)$$

where,

z' = vertical axis (dimensional soil depth)

y' = is horizontal axis (considered to be infinite).

t' = time (written in dimensional format)

w' = velocity of suction.

T' = temperature (in dimensional format),

T'_w = wall temperature

T'_∞ = temperature of free stream

ρ = density

C_p = heat capacity

k = thermal conductivity

q'_r = heat flux.

Using dimensionless factor as they were used by Mohammed (2013) and Akinpelu *et al* (2016),

$$t = \frac{t' w_0^2}{w}, \quad z = \frac{w_0 z'}{w}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$\omega = \frac{w \omega'}{w_0^2} \quad (5)$$

The suction velocity in line with Nwaigwe (2010) is selected as time-dependent and given as

$$w' = -w_0(1 + \varepsilon A e^{i\omega' t'}) \quad (6)$$

with

w_0 = suction velocity (initial)

A = suction parameter

ω = oscillation frequency

In addition, by Krishna & Reddy (2016), heat flux is written as

$$\frac{\partial q'_r}{\partial z'} = 4\tau^2(T' - T'_\infty) \quad (7)$$

τ being the absorption coefficient.

Also, Kareem and Salawu (2017), the thermal conductivity that is time-dependent is formulated as

$$k = \gamma_0(1 + \chi t) \quad (8)$$

with

α = parameter of variable thermal conductivity

t = time

γ_0 = constant thermal conductivity

Resulting from equation (2):

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial z} = \frac{1}{P_r} \left\{ \frac{\partial}{\partial z} \left((1 + \alpha t) \frac{\partial \theta}{\partial z} \right) \right\} + Q\theta - R \quad (9)$$

Subject to:

$$\theta = 1 \text{ at } z = 0 \quad (10)$$

$$\theta \rightarrow 0 \text{ as } z \rightarrow \infty \quad (11)$$

where,

$$P_r = \frac{w\rho C_p}{\gamma_0} \quad (\text{Prandtl number})$$

$$R^2 = \frac{4\tau^2\theta_w}{w_0^2} \quad (\text{the radiation parameter}),$$

and

$$Q = \frac{Q_0 w}{\rho C_p w_0^2} \quad (\text{internal heat parameter})$$

3 Method of Solution

Deploying perturbation method, Equation nine is abridged to O.D.E. This is then analytically resolved. Implicit solution for ambient temperature is written as:

$$\theta(z, t) = \theta_0 + \varepsilon e^{i\omega t} \theta_1 \quad (12)$$

Putting equation (12) alongside the derivatives into equation (9), and ignoring higher order terms $o(\varepsilon)^2$ with oversimplification, we obtain

$$\theta_0'' + \frac{P_r}{(1 + \chi t)} \theta_0' + \frac{P_r Q}{(1 + \chi t)} \theta_0 = \frac{P_r R^2}{(1 + \chi t)} \quad (13)$$

$$\theta_1'' + \frac{P_r}{(1 + \chi t)} \theta_1' + \frac{P_r(Q - i\omega)}{(1 + \chi t)} \theta_1 = \frac{-P_r A}{(1 + \chi t)} \theta_0 \quad (14)$$

primes, with respect to z represent ordinary differentiation.

Using the assumed solution (12), boundary conditions (10) and (11) can be rewritten as follows:

$$\theta_0 = 1, \quad \theta_1 = 0 \quad \text{on} \quad z = 0 \quad (15)$$

$$\theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty \quad (16)$$

Resolving equations (13) – (14) subjected to boundary conditions represented by (15) – (16), resulted transient temperature distribution is:

$$\theta_0 = C_1 e^{m_1 z} + C_2 e^{m_2 z} + C_3 \quad (17)$$

$$\theta_1 = C_4 e^{m_3 z} + C_5 e^{m_4 z} + C_6 e^{m_1 z} + C_7 e^{m_2 z} \quad (18)$$

With the assumed solution, the ground temperature becomes,

$$\theta = (C_1 + C_6 \varepsilon e^{i\omega t}) e^{m_1 z} + (C_2 + C_7 \varepsilon e^{i\omega t}) e^{m_2 z} + \varepsilon e^{i\omega t} (C_4 e^{m_3 z} + C_5 e^{m_4 z}) + C_3 \quad (19)$$

4. Results and Discussion

Results of ambient temperature was computed numerically & displayed on graphs. The Solar radiation parameter, primordial heat (Q) & Prandtl number (Pr) were inspected on Temperature gradient of both the unsaturated and saturated sand.

Table 1 shows the thermal conductivities values for both the unsaturated and saturated sandy soils.

Table 1: Thermophysical properties of saturated sandy and loamy soils

Soil	Wet Basis (%)	Thermal Conductivity (W/m K)
Sand	1.4	0.58
Loam	1.4	0.29

Except stated otherwise, the below default values were incorporated.

$$Q = 0.01, P_r = 0.71, \varepsilon = 0.01, \omega = \frac{\pi}{2}, t = 0.1, A = 0.5, A_0 = 1, R = 0.1$$

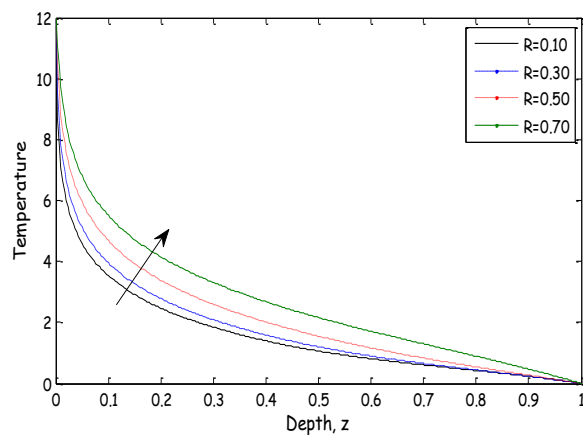


Figure 1: Solar radiation Impact on saturated Sandy soil temperature

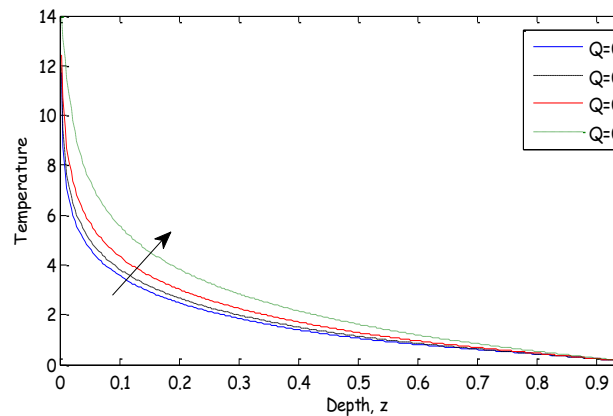


Figure 2: Impact of primordial heat on the sandy soil temperature

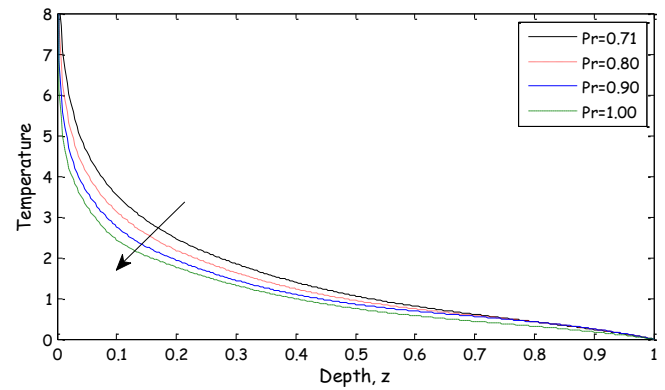


Figure 3: Impact of Prandtl number on the Sandy soil temperature

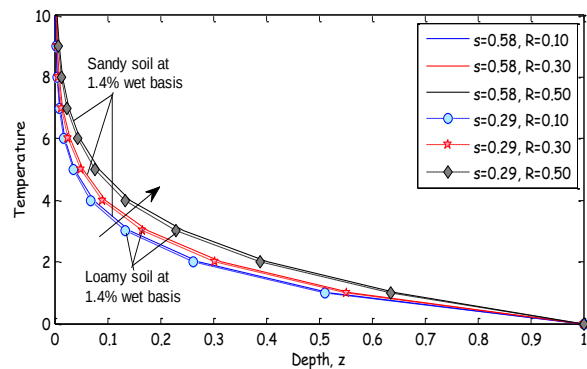


Figure 4: Comparison of solar radiation effect on temperature of Sandy soil and Loamy soils at the same moisture content

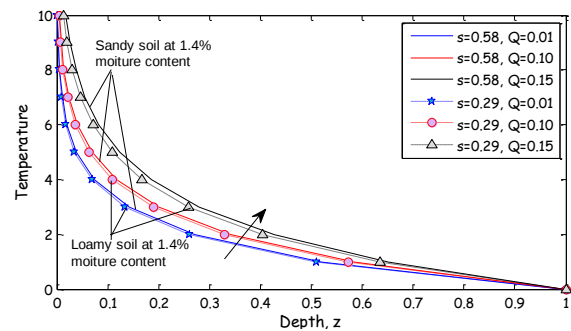




Figure 5: Comparison effect of Internal heat on the temperature of Sandy soil and Loamy soils at the same moisture content

5. Conclusion

The study of the comparison of the impact of some basic factors influencing temperature of saturated sandy & loamy soils was done. It was revealed that a boost in solar radiation & primordial heat will generally heighten the soil's temperature. But enhancement in Prandtl number lowers the temperature. Moreover, height of increase in the temperature of sandy soil is slightly more than that of loamy soil but the increase occurs almost at the same rate.

6. References

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Appendix

$$m_1 = -\frac{P_r}{2(1+\chi t)} + \sqrt{\frac{P_r^2}{4(1+\chi t)^2} - \frac{P_r Q}{1+\chi t}}$$

$$m_2 = -\left(\frac{P_r}{2(1+\chi t)} + \sqrt{\frac{P_r^2}{4(1+\chi t)^2} - \frac{P_r Q}{1+\chi t}} \right)$$

$$m_3 = -\frac{P_r}{2(1+\chi t)} + \sqrt{\frac{P_r^2}{4(1+\chi t)^2} - \frac{P_r Q}{1+\chi t} + \frac{P_r i\omega}{1+\chi t}}$$

$$m_4 = -\left(\frac{P_r}{2(1+\chi t)} + \sqrt{\frac{P_r^2}{4(1+\chi t)^2} - \frac{P_r Q}{1+\chi t} + \frac{P_r i\omega}{1+\chi t}} \right)$$

$$C_1 = -C_3 e^{-m_1 z}$$

$$C_2 = 1 + C_3 (e^{-m_1 z} - 1)$$

$$C_3 = R^2 / Q$$

$$C_4 = \frac{-C_6 e^{m_1 z}}{e^{m_3 z}}$$

$$C_5 = -(C_4 + C_6 + C_7)$$

$$C_6 = \frac{-P_r A m_1 C_1}{m_1^2 + s t m_1^2 + P_r m_1 + P_r Q - P_r i\omega}$$

$$C_7 = \frac{-P_r A m_2 C_2}{m_2^2 + s t m_2^2 + P_r m_2 + P_r Q - P_r i\omega}$$